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Determination of the unsaturated hydraulic diffusivity of porous construction materials from transient moisture profiles utilizing pin-type resistance sensor array

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The research into moisture transport is important in various science areas such as soil science and agriculture, construction and chemical engineering [1].

It is well established that the extended Darcy's Equation [2], namely:

$$\frac{\partial u}{\partial t} = \frac{\partial}{\partial x}\left(D(u)\frac{\partial u}{\partial x}\right) \qquad (1)$$

can be applied to deal with unsaturated flows such as capillary absorption of water and other liquid in porous building materials with relative success [1, 3–7]. The unsaturated hydraulic diffusivity, $D(u)$, is highly dependent on moisture content or saturation level because of the high dependence of capillary pressure on moisture content [8]. If the diffusivity as a function of moisture content is known for any particular materials, the moisture migration properties can be completely predicted by Equation (1) by combining appropriate initial condition and boundary conditions of the problem. So, the diffusivity function plays a key role in the simulation of moisture transport processes by use of the extended Darcy's equation. Traditionally, the diffusivity was obtained by assuming it to be an approximately exponential function of water content [1, 3, 4, 7], $D = D_0 \exp(\beta$ u), where D_0 and β are constants, which depend on the material. This implies that the logarithm of diffusivity il D has a linear relation with moisture content, u. If the Boltzmann transform $\lambda = x/t^{\ 1/2}$ is applied to experimental transient moisture profiles, the experimental master curve of u versus λ can be plotted. The diffusivity in an exponential format can then be determined by comparing the simulated master curve with that obtained from experimental transient profiles. However, the experimental results in [4] and those summarised in this letter indicate strong deviations from a linear relationship for small moisture contents (see Fig. 3). The present work of the authors aims to obtain a full range diffusivity function or an universal diffusivity function for unsaturated absorption by employing advanced curve fitting techniques.

The transient moisture profiles and associated master curves were obtained by utilizing a 20 channel real-time automatic computer monitoring system for moisture-profiles developed on the basis of a pin-type, resistance measuring sensor array [9]. The materials investigated were three types of commercial thistle gypsum plaster materials manufactured in UK: Plaster of paris (PP), multifinish plaster (MP) and universal one-coat plaster (UOP). Cuboid samples of the above materials were prepared by using a gang mould, size 20 × 20 × 100 mm, with predetermined water/plaster ratio and effective porosity. Immediately following the casting of samples, 20 pairs of pin-type electrodes were evenly embedded along the sample to form a one-dimension sensor array. The samples were then put in an oven for drying at temperature 38 ± 0.5 C until constant weight. The electrode sensors are stainless steel, diameter 1 mm and length 30 mm. The electrodes are electrically insulated from surrounding materials by heat shrink sleeving and leave a 5 mm tip of electrode in contact with materials. The resistance as a function of moisture content for gypsum plaster materials were previously calibrated [8]. This enabled the moisture profiles to be obtained by scanning the resistances across electrode-pairs by the monitoring system previously developed by Wang *et al.* [9]. In the measuring process, one end of the sample is in contact with water where the absorption is from the dry state. The sidewalls and the far end of sample were all sealed by polythene film to ensure one-dimension absorption. By scanning the resistance of electrode-pairs at programmable time intervals, the moisture profiles as function of time were obtained. Fig. 1 shows the moisture profiles as function of time of plaster of paris material where u_r

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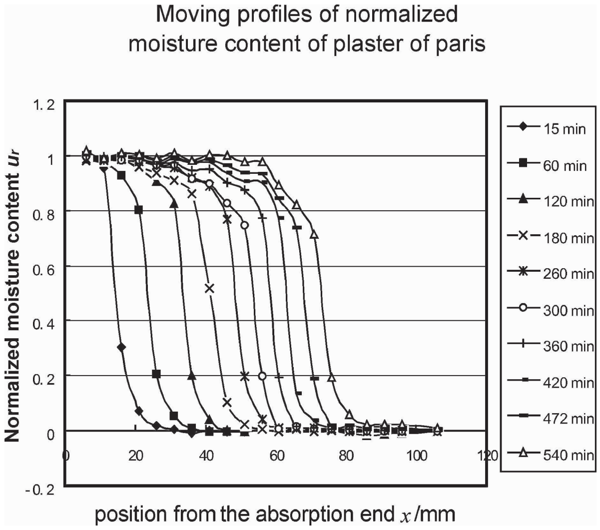

Figure 1 Moisture profiles for plaster of paris materials in water absorption processes.

is the normalized moisture content defined as:

$$u_r = \frac{u_m - u_0}{u_{\max} - u_0} \tag{2}$$

where u_m is the moisture content by mass, u_0 and $u_{\max}$ are the minimum and maximum moisture content attained in a particular process.

The moisture profiles in Fig. 1 shows that the capillary absorption processes display apparently diffusive feature and imply the possibility that diffusion type equations may be applied to describe the absorption processes. By application of the Boltzmann transformation, $\lambda = x/\sqrt{t}$, to the moisture profiles of PP (in Fig. 1), MP and UOP, the master curves showing $u_r - \lambda$ dependences of three types of gypsum plaster materials were obtained as shown in Fig. 2.

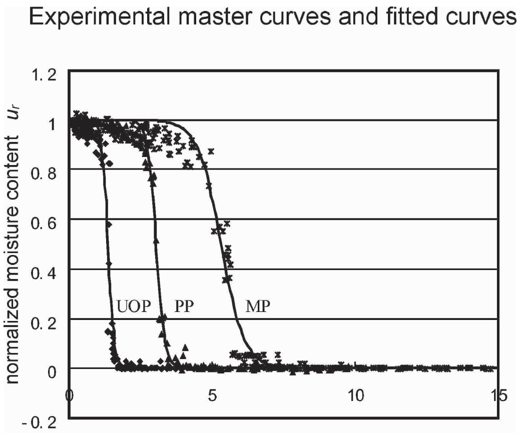

Figure 2 Master curves of three types of plasters: experimental data and regression curve. The solid lines are fitted curves of experimental points.

TABLE I The *a*, *b* values and correlation coefficients

Parameter	Plaster of paris (PP)	Universal one coat plaster (UOP)	Multifinsh plaster (MP)
a	3.048	1.359	5.315
b	–2.889	–4.066	–1.084
R^2	0.9928	0.9675	0.9844

To obtain diffusivity as a function of moisture content over full range of moisture content from the experimental master curve, here we first find a function of $u_r = u_r(\lambda)$ by regression of master curves. The analytical diffusivity function can then be obtained by using the derived Equation (6) for $D(u)$. The curve fitting for master curves were performed utilizing professional curve fitting software, GraphPad Prism, Prism 4 for Windows, version 4.01 (Trial, 2004), developed by GraphPad software Inc. (USA). Non-linear regression techniques were applied with constraints of u_r max $= 1$, $u_{r0} = 0$. After trials of different fitting functions for their shape and errors, the *basic dose response curve* function was observed to have the highest correlation coefficient (as shown in Table I) and was chosen as the best function describing master curves. It can be expressed as:

$$u = \frac{1}{1 + 10^{(a-\lambda)b}} \tag{3}$$

where a, b are undetermined parameters. The a, b values for different gypsum materials are summarized in Table I.

If the Boltzmann transformation is applied, the non-linear moisture transport Equation 1 can be reduced to the ordinary diffusion Equation (ODE)

$$2\frac{d}{d\lambda}\left(D(u)\frac{du}{d\lambda}\right) + \lambda\frac{du}{d\lambda} = 0 \tag{4}$$

Considering the boundary and initial condition:

$$\begin{aligned} u &= u_s \text{ at } \lambda = 0; \\ u &= u_0 \text{ for } \lambda \to \infty \end{aligned} \tag{5}$$

Equation 4 with boundary conditions (5) has only one solution. So the profiles at different times are related by a simple $\sqrt{t}$ scaling. The diffusivity $D(u)$ for absorption can then be determined by integrating Equation 4 with respect to λ using the boundary condition (5) [3, 4]:

$$D_u = -\frac{1}{2\left(\frac{du}{d\lambda}\right)\Big|_u}\int_{u0}^{u}\lambda \, du' \tag{6}$$

Differentiation of (3) (by Matlab) yields the differential F:

$$F = \frac{du}{d\lambda}|_u = \frac{1}{(1 + 10^{(a-\lambda)b})^2}10^{(a-\lambda)b} b \cdot \ln 10 \tag{7}$$

From function (3), λ can also be derived, giving:

$$\lambda = a - \frac{\ln((1-u)/u)}{b \cdot \ln 10} \tag{8}$$

TABLE II The $u_{\min}$ values of three types of gypsum materials

	PP	UOP	MP
$u_{\min}$	0.07	0.08	0.08

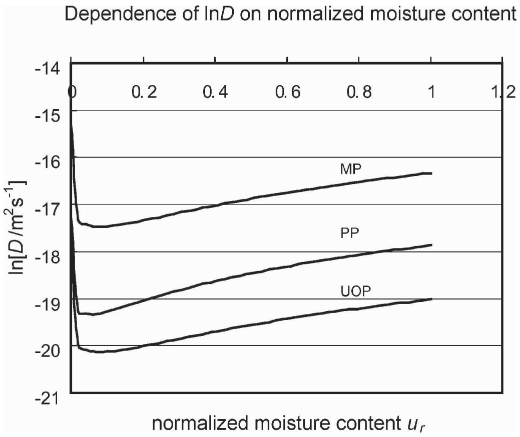

Figure 3 Dependence of logarithm Diffusivity (ln$D$) of gypsum plasters as function of normalized moisture content for three type of gypsum materials.

Integrating (8) from 0 to u with respect to u, gives the integral G:

$$G = \int_0^u \lambda du' \tag{9}$$

$$= 1/P_1\{P_1 \ \mathrm{a} \cdot \mathrm{u} \cdot \mathrm{b} - P_2 \cdot \ln(1/u)$$

$$+P_2 \cdot \ln[(1-u)/u] - P_2 \cdot \ln[(1-u)/u] \cdot u\}/b$$

where $P_1 = 2.592 \times 10^{15}$, $P_2 = 1.126 \times 10^{15}$. The diffusivity (6) can be expressed in terms of F and G as:

$$D_u = -\frac{G}{2F} \tag{10}$$

Combining Equation 7 to Equation 10 with values of a, b in Table I, the diffusivity function $D(u)$ can be determined.

Fig. 3 shows the dependence of diffusivity on normalized moisture content of three types of plaster materials. It strongly indicates that the logarithm of diffusivity shows an approximately linear relation with moisture content in most of moisture content range, but fails in the small moisture content range. The diffusivity dramatically increases when moisture content becomes lower then the point $u_{\min}$., defined as the lowest point of the curve. The increase is possibly caused by the fact that the capillary pressure will significantly increase when u_r becomes lower [8]. Table II lists the values obtained for three gypsum materials. The results indicate that the assumption of exponential format of the dependence of diffusivity on moisture content, which is commonly used in research of capillary absorption processes for porous materials as in reference [1, 3–7], is not valid in small value range of moisture content. The more general function describing the dependence of diffusivity on moisture content, as shown by Equations 7–10, was determined by non-linear regression of master curves. The obtained diffusivity function is extremely useful in the simulation of the moisture transport processes now being investigated.

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